4

Delivery Systems and Initial Dilution

Robert C.Y. Koh

From Ocean Disposal of Municipal Wastewater: Impacts on the Coastal Environment, Volume 1. E.P. Myers, ed. Cambridge, Mass: Sea Grant College Program, M.I.T., 1983.

INTRODUCTION

The discharge of municipal waste into the ocean has been practiced for many decades by coastal communities. There are basically two methods of transporting the waste from the treatment works to the discharge point: 1. via an outfall pipeline for effluent or sludge, and 2. for sludge, by means of barges or vessels.

In the continental United States, there are two major groups of ocean discharges of municipal waste. (Chapter 1 summarizes the total extent of municipal discharges for the nation.) On the Atlantic coast, there is a major assemblage of ocean discharges from both outfalls and barges into the area known as the New York Bight. On the Pacific coast, there is another major cluster of ocean discharges from outfalls into the Southern California Bight. While there are other discharges along most shorelines, these two clusters not only are the largest, but also the most studied. Extensive and detailed measurements from these two areas will undoubtedly shed much light as to the effects of ocean discharges of municipal wastes. Table 4.1 summarizes the characteristics of selected major outfalls along the Pacific coast and Hawaii. The discharges into the New York Bight are more diffuse and have been documented in Gross (1976) and Mueller and Anderson (1978). The appendices at the end of this book contain more detailed descriptions of these and a few other areas.

Also included in Table 4.1 are costs and dates construction contracts were awarded. Construction costs of marine outfalls not only vary from one region to another, but also depend on size of pipeline, construction method, wave protection features, and special structures required, such as Y-structures to diffuse the flow into two legs.

Table 4.1 Summary of Characteristics of Major Pacific Ocean Outfalls (USA). (Notes on following page)

	White Point No. 3[1]	Hyperion[2]	San Diego	White Point No. 4[3]	West Point[4]	Orange County[5]	Sand Island[6]
Year Operation began	1956	1960	1963	1965	1965	1971	1975
Pipe diameter (inside)-(inches)	90	144	108	120	96	120	84
Length of Main Outfall (excl. diff.)-(ft)	7,900	27,525	11,500	7,440	3,050	21,400	9,120
Length of Diffuser L_d (ft)	2,400	7,920	2,688	4,440	600	6,000	3,384
Depth of Discharge (nominal)	200-210	195	200-210	165-190	210-240	175-195	220-235
Design average Flow Q (ft^3/sec)	232	651	363	341	194	450	164
Port diameters[a] (inches)	6.5-7.5	6.75-8.13	8.0-9.0[b]	2.0-3.6	4.5-5.75	2.96-4.13	3.00-3.53
Port spacing (average)[c]-(ft)	24	48	48	6	3	12	12
Velocity of Disch. (nominal) for ave. flow -(fps)	8	13	15	9	6	13	10
Q/L 2 d (ft/sec)	0.097	0.082	0.135	0.077	0.323	0.075	0.048
Area Factor (Total Port Area/Pipe Area)	0.63	0.44	0.39	0.51	0.60	0.45	0.44
Date Contract Awarded	10/54	5/57	9/61	5/64	7/64	10/15/68	10/73
Construction Cost (Million $)	2.18	20.21	7.78	4.46	1.23	8.95	13.57

[a] Exclusive of end ports, which are usually somewhat larger.

[b] Blocked by orifice plates with openings of 6.5-7 inches for early years' low flow.

[c] Length of diffuser divided by number of ports; real spacings on each side of the pipe are twice the values indicated.

[1] Sanitation Districts of Los Angeles County Whites Point No. 3
[2] City of Los Angeles at Hyperion
[3] Sanitation Districts of Los Angeles County White Point No. 4
[4] Metrop. Seattle (West Point)
[5] Sanitation Districts of Orange County, CA.
[6] Honolulu (Sand Island)

Along the southern California coast, the offshore bathymetry is relatively steep (slopes are of order 1 on 100) so that a water depth of 60 m is attained at a distance of about 6 km from shore. Outfall pipes often reach beyond that distance and discharge into water depths in excess of 60 m. Figure 4.1 shows the plan and profile of the Sand Island outfall system for the city and county of Honolulu, Hawaii. Equipped with long diffusers, these outfall structures result in wastewater plumes which attain initial dilutions on the order of 10^2 to 10^3. In contrast, the shelf slopes offshore of New York-New Jersey are much flatter, on the order of 1 in 500. To reach an equivalent depth would therefore require a distance five times as long.

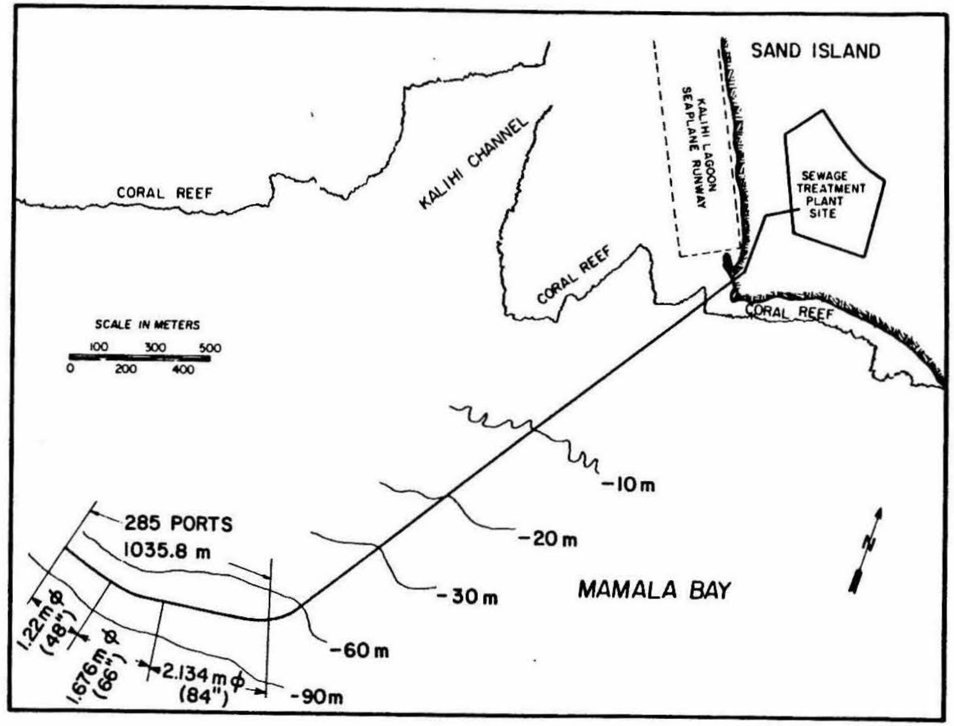
Traditionally, New York has used barges to dispose of solid wastes, including sewage sludge, in the ocean. These disposal operations take place in designated dumping sites. After 1934, when the ocean dumping of refuse, garbage, and floatable waste was stopped, the types of waste discharged from barges have included sewage sludge, dredge spoil, cellar dirt, and industrial waste, with dredge spoil accounting for the largest fraction.

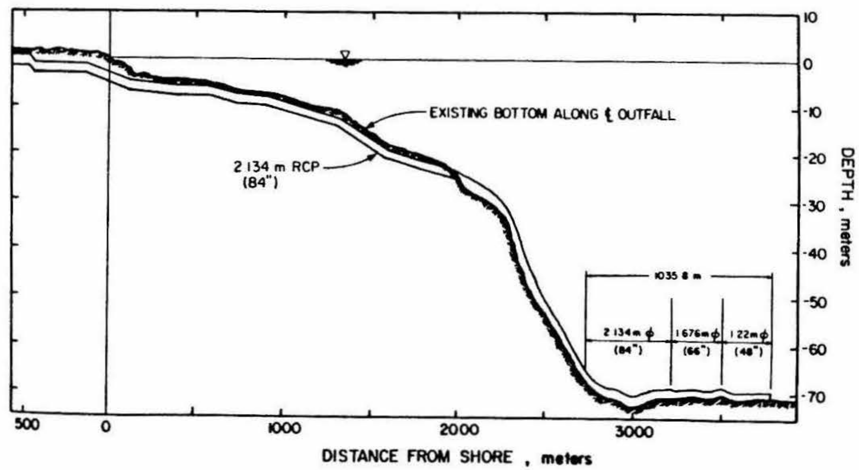
The purpose of this chapter is primarily to describe the physical mixing processes which occur when waste is discharged into the ocean via either outfall or barge. In chapter sections, the general physical mechanisms are discussed; the concept of initial dilution is described; results of research conducted in the past two decades which permit estimation of initial dilutions as a result of ocean waste disposal are presented; marine environmental conditions influencing mixing processes are discussed; and the chapter is summarized.

MIXING PROCESSES

Sewage effluent is primarily fresh water. When discharged from an outfall into the ocean, it rises due to its buoyancy. During the buoyant rise, it mixes with the ambient ocean water. The density of the mixture thus gradually increases as the diluted effluent contains progressively more seawater. If the ocean water is density stratified, as is often the case in summer due to presence of a thermocline, the diluted effluent may stop rising before it reaches the surface. This occurs if the effluent is mixed

Figure 4.1 Plan and profile of Sand Island Outfall (R.M. Towill Corp., 1972)





sufficiently with the heavier water below the thermocline such that the mixture density is greater than that of the water above the thermocline.

Sewage sludge is also primarily fresh water. Digested sludge, which contains several percent solids, is normally lighter than seawater so that, if discharged into the ocean, it tends to rise towards the surface. For discharge from an outfall, the sludge from the city of Los Angeles is prediluted with two parts effluent to reduce the risk of deposition in the pipeline. For discharge from barges, the sludge can be thickened to reduce its bulk. This also tends to render it heavier than seawater.

The initial behavior of sludge discharged from a barge depends on both the method of discharge and on the sludge characteristics. When heavier-than-seawater sludge is pumped from a barge, the resulting plume would sink much like the buoyant plume would rise from an outfall. If a bottom opening hopper barge is used, the sludge, if thickened to the point of cohesion, may fall in a unit without much dilution with the receiving water, like a lightweight rock. During this time, pieces may ablate from the mass. A more typical occurrence is when it falls as a loose cloud of solid particles in water. In this case, its physical behavior is similar to that of the rise of a thermal in the atmosphere. Mixing with the ambient water occurs while the sludge cloud still retains its identity. From the physical point of view, the solid particles in the sludge need not be considered except for their contribution to the density of the descending sludge cloud, because the settling velocity of individual particles is several orders of magnitude less than the advective velocity of the cloud. If the ocean water is density stratified, the descending sludge cloud may not reach the bottom but spread out horizontally in a layer of neutral density. The solids in the sludge would gradually settle out.

The initial phase of motion is thus strongly influenced by the difference in density between the waste cloud (or plume) and the ambient water. It is interesting to point out that this phase of motion occurs in a matter of minutes rather than hours. The extent of mixing accomplished during this phase is generally accepted as initial dilution. Since it is the only phase under the control of the design engineer, it has received the most attention both in research studies and regulatory posture.

Ocean current is another variable which affects the behavior of the discharge plume. The plume tends to bend over in the direction of the current and the effectiveness of mixing is generally greater when there is a current. However, it is usual in typical coastal waters to have periods of very low or effectively zero current. Since initial mixing occurs in minutes, design calculations of initial dilution are often based on zero current.

Figure 4.2 illustrates the behavior of the wastewater plumes under these various ambient conditions and discharge methods.

Following the buoyant transport, the diluted waste material would be in one of three vertical locations: 1. in a layer of neutral buoyancy, 2. on the bottom, where the diluted waste material is heavier than the local seawater, or 3. at the surface, where the diluted waste is lighter than the surface seawater. The next phase of motion of the waste cloud is horizontal spreading. The driving force in this case is either the residual density difference between the diluted waste and the local seawater or the difference in vertical density gradient within and outside the waste field. This spreading is resisted by fluid inertia, pressure gradients, interfacial shearing which may include some mixing, and possibly wind, if on the surface, or bottom friction, if on the bottom.

Both the buoyant ascent or descent and the horizontal spreading occur over relatively short periods of time, from minutes to an hour. Subsequent to those, the diluted waste field is dynamically passive and further mixing and transport are dominated by ocean currents and turbulence. This part of the mixing process is treated in Chapter 5.

INITIAL DILUTION

The term dilution, as has been used in the literature on waste disposal, denotes the reciprocal of the volume concentration of discharged waste in the receiving water (Rawn and Palmer, 1930). Thus, if c is volume concentration, and S is dilution, then

$$S = \frac{1}{c} = \frac{\text{total volume of a sample}}{\text{volume of discharged waste in the sample}}$$

and S = 1 for an undiluted sample

Figure 4.2a Schematic of wastewater plume behavior for various ocean conditions and discharge methods.

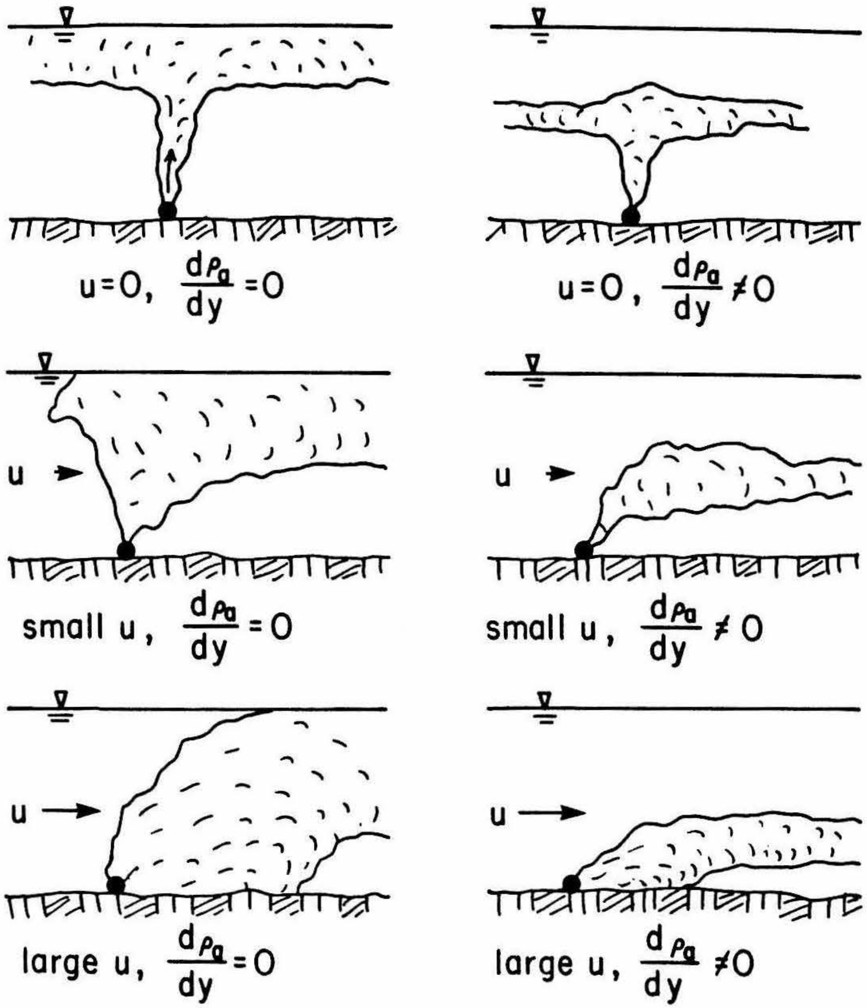
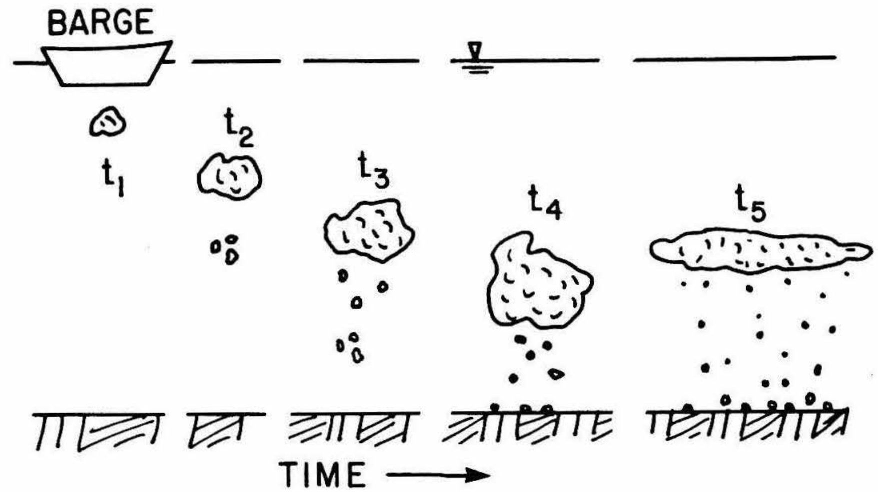
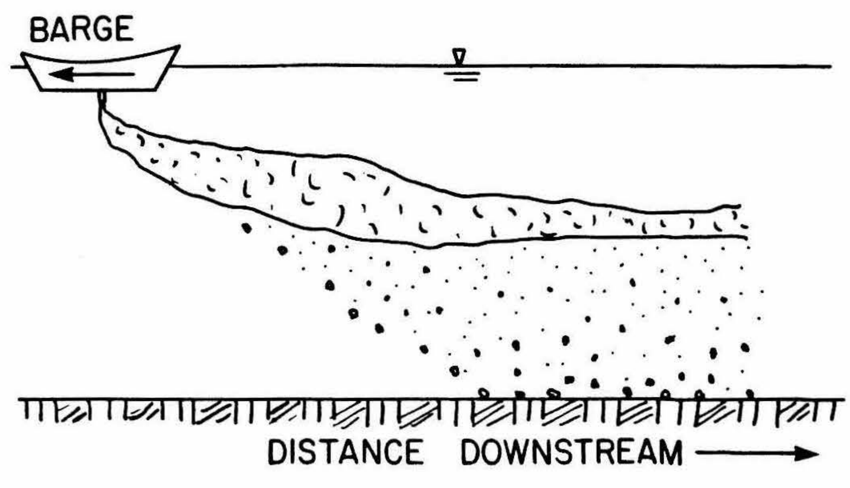


Figure 4.2b Schematic of waste behavior for discharge from barge.



Various stages after heavier-than-seawater sludge is discharged instantaneously from a bottom opening hopper barge.



To the extent that it is possible to take as small a sample as desired, one can define dilution at a point. In general, dilution is a function of space and time and can take on any value larger than or equal to unity. It should be noted that a dilution of 10, as used herein, implies the mixture of 9 parts of water with 1 part wastewater, not 10 parts to 1 part, as sometimes used by others. The primary reason for using dilution rather than concentration is probably a matter of convenience.

Initial dilution for wastewater discharged through an outfall denotes the dilution resulting from mixing which occurs during the buoyant-rise phase of the plume, dominated by the buoyancy and momentum fluxes at the point of discharge. For discharge of heavier-than-seawater sludge from a barge, it is also possible to designate as initial dilution the mixing which results from the sinking of the waste cloud either to the bottom or to its level of neutral buoyancy. The mixing which results after the plume rise or fall is sometimes referred to as subsequent dilution, or further dilution. The term "initial dilution" is not precisely defined, since not only is it difficult to mark the point where initial dilution ends and subsequent dilution begins, but also sometimes it is not even possible to designate a phase of motion as strongly influenced by the discharge momentum and buoyancy. For example, if a slightly lighter-than-seawater sludge is discharged at the surface from a barge, no sinking plume may be manifested since the waste would simply spread out on the surface.

Dilution, as defined by the reciprocal of the volume concentration, is a function of both position and time. Within a steady rising plume, the dilution at a fixed location varies with time. It is possible to define a time-averaged dilution $\bar{S}$ by

$$\bar{S} = \frac{1}{\bar{c}}$$

where $\bar{c}$ is the time-averaged volume concentration. $\bar{S}$ is then a function only of position, and hence varies across the plume cross-section. It has generally been found that a minimum in $\bar{S}$ occurs at the center of the plume cross-section. This value of $\bar{S}$ is commonly referred to as centerline dilution, which, of course, still varies with distance along the plume. In addition to centerline dilution, there is another

often utilized notion for a measure of mixing, viz. that of average dilution. It should be noted again that dilution values should not be averaged. Rather the averaging should be performed on the concentration values and the reciprocal taken on the result. The term average dilution, as commonly used in waste disposal, refers to an averaging process based on the flux of the waste material. For this reason, it is sometimes more explicitly referenced as flux-weighted-average dilution. Thus, if c and u are the time-averaged concentration and axial velocity in the plume, then the flux-weighted average dilution S_a is defined as

$$S_a = \frac{\int_A u\,dA}{\int_A c\,u\,dA}$$

where A is a plane normal to the plume axis.

The above definitions of centerline and average dilutions apply to a plume such as is formed above an outfall. These definitions are the commonly accepted ones found in the literature on waste disposal. For discharge from a vessel such as a barge where the waste load may be released in a short time period rather than continuously, the concepts of centerline dilution and average dilution become less well defined. Dilution can still be defined as the reciprocal of the volume concentration. However, ensemble averaging is required in place of time averaging to arrive at readily interpretable dilution values. This case is handled in the next section where physical phenomena are examined.

In this section, the concepts of dilution for buoyant jets have been defined, including the commonly used terms, initial dilution, centerline dilution, and average dilution. This is done primarily because regulatory documents make specific reference to these terms and because they are sometimes misinterpreted.

ESTIMATION OF INITIAL DILUTIONS

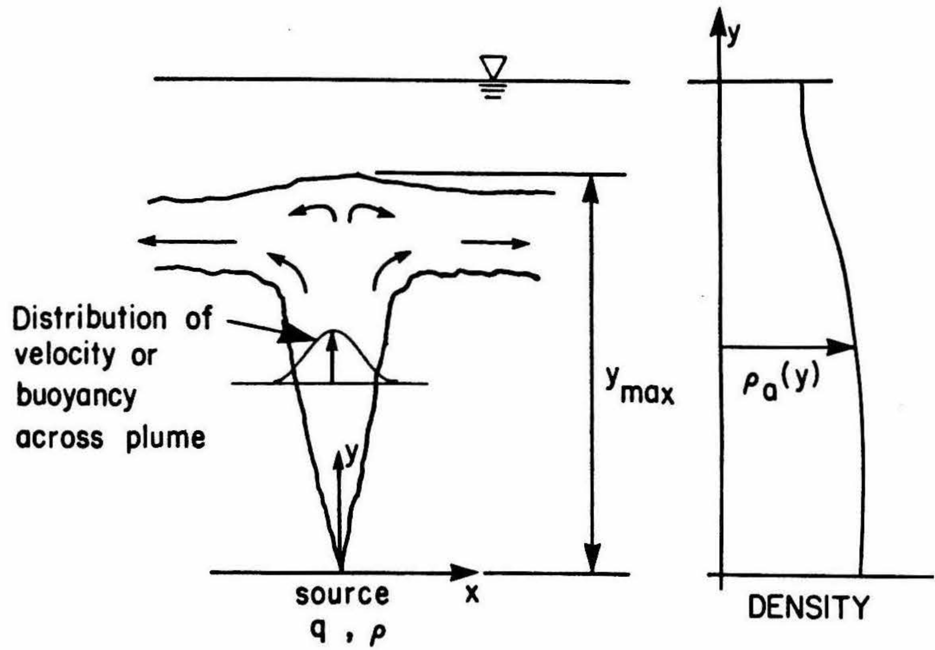
Initial dilution results from that part of mixing which occurs due to the momentum and buoyancy of the discharge and represents the only part under the control of the designer of the discharge method. In this section, the results of research directed at development of methods to estimate initial dilution will be discussed.

Historically, the calculation of primary interest to sanitary engineers involved with ocean sewage outfalls has been the area of ocean pollution as indicated by bacteriological surveys. Prediction of such areas was based largely on a factor which gives the number of "acres-per-second-foot-of-sewage." ("Second-foot" is an old engineering unit synonymous with cubic-feet-per-second.) This factor was used generally as a constant independent of the particular design. The first attempt to investigate the mixing characteristics which result from the discharge of sewage effluent into seawater was probably by Rawn and Palmer (1930) who performed small scale experiments in the sea. Dyed fresh water was discharged through submerged nozzles and dilutions measured by color comparisons. However, it was not until 1956 that the analysis of mixing from buoyant jets and buoyant elements was put on a sound theoretical basis by Morton, et al. (1956). In that classic paper, they presented analyses for both continuous and instantaneous point sources of mass, momentum, and buoyancy fluxes. The former is related to the discharge of buoyant effluent from outfalls, while the latter is related to the discharge of heavier-than-seawater sludge from hopper barges. Subsequently, there have been a large number of investigations on the subject of buoyant jets and plumes (fewer on instantaneous sources of buoyant elements). The methods as well as results may differ in form but not in substance. Summaries of this type of research on buoyant jets and plumes are found in Brooks (1973), Koh and Brooks (1975), and Fischer et al. (1979).

Continuous Source

Consider a steady, continuous line source of mass, momentum, and buoyancy fluxes along the z-axis of a rectangular coordinate system, as shown in Figure 4.3. The case for a point source can be analyzed similarly (see Morton et al., 1956). The line source is chosen here because many large outfalls are equipped with long multiple-port diffusers which can be approximated by line sources.

Figure 4.3 Definition sketch of line plume in a stratified environment.



Assume that:

1. the ambient fluid is motionless (except for the motion induced by the plume);
2. the ambient fluid is infinite in extent and stably stratified in density;
3. all density changes are small (so that density changes need only be considered when associated with gravity) and are linearly related to the agent causing the stratification;
4. flow in the plume is fully turbulent; and
5. pressure distribution is hydrostatic.

The integral form of the conservation equations can be written as follows:

Conservation of mass flux

$$\frac{d}{dy}\left\{ \int_{-\infty}^{\infty} \rho^* u^* dx \right\} = E\rho_a \tag{4.1}$$

where ρ^*, u^* are the density and axial velocity, E is the rate of volume entrainment and ρ_a is the density of the ambient water being entrained.

Conservation of momentum flux

$$\frac{d}{dy}\left\{ \int_{-\infty}^{\infty} \rho^* u^{*2} dx \right\} = g \int_{-\infty}^{\infty} (\rho_a - \rho^*) dx \tag{4.2}$$

were g is gravitational acceleration.

Conservation of buoyancy flux

$$\frac{d}{dy}\left\{ g \int_{-\infty}^{\infty} (\rho_1 - \rho^*)\ u^* dx \right\} = Eg\ (\rho_1 - \rho_a) \tag{4.3}$$

where $\rho_1 = \rho_a(0)$.

The equation 4.3 has made use of assumption 3. The same assumption also permits the replacement of ρ^* by ρ_1 (a typical constant value) in the left hand side of equations 4.1 and 4.2.

Rather than dealing with ρ^*, it will be convenient to introduce the variable Δ^* defined by

$$\Delta^* = g\, \frac{\rho_a - \rho^*}{\rho_1} \tag{4.4}$$

Equations 4.1 through 4.3 can now be written

$$\frac{d}{dy}\left\{ \int_{-\infty}^{\infty} u^* dx \right\} = E \tag{4.5}$$

$$\frac{d}{dy}\left\{ \int_{-\infty}^{\infty} u^{*2} dx \right\} = \int_{-\infty}^{\infty} \Delta^* dx \tag{4.6}$$

$$\frac{d}{dy}\left\{ \int_{-\infty}^{\infty} \Delta^* u^* dx \right\} = \frac{g}{\rho_1}\ \frac{d\rho_a}{dy} \int_{\infty}^{\infty} u^* dx \tag{4.7}$$

The quantities u^*, and Δ^* are functions of x, y and time. They may each be decomposed into the time averaged part ($\bar{u}$ and $\bar{\Delta}$) and the fluctuating part (u' and Δ'). Thus

$$u^* = \bar{u} + u' \tag{4.8}$$

$$\Delta^* = \bar{\Delta} + \Delta' \tag{4.9}$$

Substituting into equations 4.5 to 4.7 and taking time averages of the resulting equations gives

$$\frac{d}{dy}\left\{ \int_{-\infty}^{\infty} \bar{u} dx \right\} = E \tag{4.10}$$

$$\frac{d}{dy}\left\{ \int_{-\infty}^{\infty} (\bar{u}^2 + \overline{u'^2}) dx \right\} = \int_{-\infty}^{\infty} \bar{\Delta} dx \tag{4.11}$$

$$\frac{d}{dy}\left\{\int_{-\infty}^{\infty}(\bar{\Delta}\,\bar{u} + \overline{\Delta' u'})dx\right\} = \frac{g}{\rho_1}\frac{d\rho_a}{dy}\int_{-\infty}^{\infty}\bar{u}dx \tag{4.12}$$

where the overbar denotes time averages. For closure, it is now necessary to specify $\int_{-\infty}^{\infty}\overline{u'^2}dx$, $\int_{-\infty}^{\infty}\overline{\Delta' u'}dx$, and E. The customary closure assumptions are to neglect $\int_{-\infty}^{\infty}\overline{u'^2}dx$ and $\int_{-\infty}^{\infty}\overline{\Delta' u'}dx$, and to assume that E is proportional to $\bar{u}(x=0)$ with a proportionality constant to be determined by experiments. For the line plume, E is found to be 0.28 $\bar{u}(x=0)$ based on experiments by Rouse et al. (1952).

Before proceeding further, it is well to examine in detail the above closure assumptions. The basis for the neglect of the terms $\int_{-\infty}^{\infty}\overline{u'^2}dx$ and $\int_{-\infty}^{\infty}\overline{u'\Delta'}dx$ has been primarily one of ignorance since they were difficult to measure. Measurements carried out in the laboratory have typically employed the use of some type of probe. Probes for the measurement of fluid velocity include total head tube, propeller meter, hot wire or hot film probes and more recently laser doppler velocimetry. It must now be emphasized that use of these probes for measurement may require careful interpretation. For example, suppose a Pitot tube in combination with a pressure transducer is used to measure velocity. Thus, the basic measurement is not of velocity but of dynamic pressure which is proportional to the square of the velocity. The measured "average velocity" may not be $\bar{u}$ as in equation 4.8 but rather related to the root-mean-square of u*, the exact relationship being dependent on the detailed nature of u* as well as the response of the instrument. Detailed measurements by Kotsovinos (1975) using laser doppler velocimetry combined with a fast response thermistor for the two-dimensional buoyant plume indicate that the quantities $\int_{-\infty}^{\infty}\overline{u'^2}dx$ and $\int_{-\infty}^{\infty}\overline{u'\Delta'}dx$ may not be negligible at all. Similar measurements by George, et al. (1977) for the axisymmetric buoyant plume also show that turbulent transport should not be ignored.

From the above discussions, it is seen that there may be a certain amount of confusion in the

interpretation of experimental results. On the other hand, the use of the traditional closure assumptions (neglect of the turbulent transport terms) have yielded predictions which compare favorably with experiments. In this chapter, the traditional closure assumptions will be adopted; however, the reader is cautioned that further research on the turbulent transport terms may necessitate some adjustments in the future.

Return now to equations 4.10, 4.11 and 4.12, and neglect the turbulent transport. From experiments by Rouse et al. (1952), and confirmed by others, e.g., Kotsovinos (1975), it was found that the crossplumes profiles of $\bar{u}$ and $\bar{\Delta}$ are similar and well approximated by Gaussians of the form

$$\bar{u} = ue^{-x^2/b^2} \tag{4.13}$$

$$\bar{\Delta} = \Delta e^{-x^2/b^2\lambda^2} \tag{4.14}$$

where b, a characteristic width of the velocity profile, is a function of y. The corresponding width of the density profile is λb. For line plumes a reasonable approximation is $\lambda = 1$, which is used in the analysis below for simplicity. Here u and Δ are the values of $\bar{u}$ and $\bar{\Delta}$ at x=0 (i.e., centerline values). Substituting equations 4.13 and 4.14 into the conservation equations, performing the integrations, yields the result

$$\frac{d(ub)}{dy} = \frac{2\alpha u}{\sqrt{\pi}} \tag{4.15}$$

$$\frac{d}{dy}(u^2 b) = \sqrt{2}\;b\Delta \tag{4.16}$$

$$\frac{d}{dy}(ub\Delta) = -\sqrt{2}\;ubG \quad \left(G = \frac{-g}{\rho_1}\frac{d\rho_a}{dy}\right) \tag{4.17}$$

where the closure relation E = $2\alpha u$ has been used. (α is known as the entrainment coefficient and the factor 2 represents the two sides of the plume.) This is a set of three equations for three unknown functions u, b, and Δ and is in the form of an initial value problem with initial conditions specified at y=0 which characterizes the source. For a given G(y) (the density gradient in the ambient fluid) and initial conditions u_o, b_o, Δ_o at y=0, the system can be readily solved numerically. The case for a point source can be investigated using the same technique and also results in an initial value problem. Computer programs have been published which utilize this type of analysis (known as the integral approach) to give dilutions along the rising buoyant jet discharging at an arbitrary angle into a motionless stably stratified fluid (e.g., round jets: Baumgartner et al., 1971; Ditmars, 1969; slot jets: Sotil, 1971; a row of equally spaced round jets which later merge to a slot jet: Koh and Fan, 1970.)

The entrainment coefficient α as used in this type of formulation was assumed by early investigations (e.g., Fan, 1967) to be a constant. Later research such as by Fox (1970) and List and Imberger (1973) have taken the view that α should depend on the local buoyant jet characteristics. In this chapter, α will be assumed constant and equal to the value appropriate for the two-dimensional plume because discharges from outfalls are nearly plumelike.

For a pure line plume (source of buoyancy flux only) in a uniform environment ($\rho_a=\rho_1$=constant), the solution is particularly simple:

$$u = \text{constant} = (g'q_o/\alpha)^{1/3} \tag{4.18}$$

$$b = \frac{2}{\sqrt{\pi}}\alpha y \tag{4.19}$$

$$\Delta = \frac{g'q_o}{\sqrt{2}\,u\alpha y} \tag{4.20}$$

where $g' = g\left(\frac{\rho_1-\rho_o}{\rho_1}\right)$ and q_o is the discharge per unit length. The centerline dilution is found from the conservation equation for the flux of a tracer quantity

$$\sqrt{\pi/2}\,cub = c_oq_o \tag{4.21}$$

i.e.,

$$S = \frac{c_o}{c} = \sqrt{2}\,\alpha^{2/3}g'^{1/3}y/q_o^{2/3} \tag{4.22}$$

For α = 0.14 (see Brooks, 1973)

$$S = 0.38\,\frac{(g'q_o)^{1/3}y}{q_o} \tag{4.23}$$

For a linearly stratified environment (G=constant) and assuming α = 0.14, the pure plume solution gives

$$S_t = 0.31\,\frac{(g'q_o)^{1/3}y_{max}}{q_o} \quad \text{(terminal centerline dilution)} \tag{4.24}$$

where

$$y_{max} = 2.84(g'q_o)^{1/3}G^{-1/2} \quad \text{(maximum height of rise)} \tag{4.25}$$

For horizontal slot discharge with values for initial parameters chosen to correspond to typical sewage outfalls, the corresponding formulae are (Brooks, 1973)

$$S_t = 0.36\,\frac{(g'q_o)^{1/3}y_{max}}{q_o} \tag{4.26}$$

$$y_{max} = 2.5(g'q_o)^{1/3}G^{-1/2} \tag{4.27}$$

For point source, the results are:

$$\left(\begin{array}{c}\text{uniform}\\\text{environment}\\(G = 0)\end{array}\right) \quad S = 0.09\,\frac{(g'Q_o)^{1/3}}{Q_o}\,y^{5/3} \tag{4.28}$$

$$\left(\begin{array}{c}\text{linear}\\\text{stratification}\\G=\text{constant}\end{array}\right): \quad S_t = 0.07\,\frac{(g'Q_o)^{1/3}}{Q_o}\,y_{max}^{5/3} \tag{4.29}$$

$$y_{max} = 4(Q_og')^{1/4}G^{-3/8} \tag{4.30}$$

corresponding to equations 4.23, 4.24, and 4.25, respectively.

All the above are for centerline dilutions. The average dilution is related to the centerline dilutions according to the integral approach by

$$S_a = \sqrt{2}\, S \quad \text{for two-dimensional or slot buoyant jets} \qquad (4.31)$$

$$S_a = 1.74\ S \quad \text{for axisymmetric buoyant jets} \qquad (4.32)$$

(corresponds to = 1.16 in Eq. 4.14)

The reader is again cautioned that all the above results are based on the integral approach where turbulent transport represented by terms of the form $\overline{u'c'}$ are ignored. While these have been used quite successfully in the design of many outfall diffusers, more recent research (Kotsovinos, 1975) has indicated that the turbulent transport may not be negligible. Fischer et al. (1979) should be consulted for a more detailed discussion.

Among the results detailed above, the ones of most engineering usefulness are those for the two-dimensional plume (i.e., equations 4.23 to 4.27). This is because a multiport diffuser is an engineering approximation to a two-dimensional or line source. The design variables of primary importance in determining the initial dilution are the overall length of the diffuser and the depth of discharge. The details of port spacing and number of ports are secondary provided that they are sufficiently numerous and close together to approximate a line source. A frequent misconception is to assume the dilution would be optimized by choosing the port spacing sufficiently large that the plumes from neighboring ports do not interfere. However, such a diffuser can almost always be improved by having more smaller ports closer together for the same overall diffuser length.

When sewage effluent is discharged continuously from an outfall, it rises and becomes progressively more diluted. Upon reaching either the ocean surface or a submerged level if there is sufficient density stratification in the ocean, it will spread out horizontally. A layer of diluted effluent is then formed above the outfall. It is clear that dilution of the effluent with uncontaminated ocean water effectively ceases above the bottom of this layer. The results of the integral analyses presented herein relate the dilution to height of travel assuming continual availability of uncontaminated ocean water. Application of these results to evaluation of outfall performance must include a determination of the

location of the bottom of the sewage field. This, in turn, requires the solution of the problem of horizontal spreading of a layer either buoyant at the surface or more homogeneous and submerged in a density stratified ocean. Thus, to obtain the expected initial dilution from a given design, the results of integral analyses should be coupled with the solution of the parallel problem of horizontal spreading. This latter phenomenon has received less investigation. It is similar in nature to that of oil spreading on water, the spread of river discharge upon entering the ocean, or the vertical collapse and horizontal spreading of a mixed region in a stratified fluid.

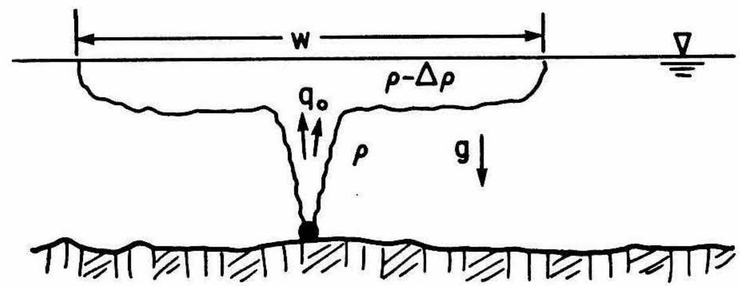


Figure 4.4 Definition sketch of surface spreading of buoyant fluid.

The spread of buoyant fluid on the surface of an otherwise stagnant fluid has been examined by Sharp (1969), Koh and Fan (1970), Koh (1976), Chen and List (1976), and Chen (1980). For a two-dimensional continuous source started at t=0, the width of the spreading field w grows with time as (see Figure 4.4)

$$w = K\ (g'q_o)^{1/3} t$$

for the time in the range of interest for application to outfalls. K was found from experiments to be about 1.2. This result can be combined with equation 4.23 to yield the initial dilution to be expected at the bottom of the spreading sewage field. This is found to be (Koh and Brooks, 1975)

$$S_b = 0.27 \frac{(g' q_o)^{1/3} d}{q_o} \quad \text{centerline dilution} \tag{4.33}$$

$$S_{ab} = 0.38 \frac{(g' q_o)^{1/3} d}{q_o} \quad \text{average dilution} \tag{4.34}$$

where d is the total depth of water. It is coincidental that the centerline dilution without considering the effect of blocking by the sewage field is the same as the average dilution including the effect of blocking.

The initial dilution obtained from typical large outfalls equipped with multiport diffusers can be readily estimated based on equation 4.33 or 4.34. Referring to Table 4.1 it can be seen that q_o is of order 0.1 cfs/ft or 0.01 m^2/s. Also $g' = 9.81 \times 0.025$ m/sec^2 where the difference in density between seawater and effluent has been assumed to be 0.025 gm/cc. For d=60 m,

$$S_b \cong 220$$

$$S_{ab} \cong 310$$

These values assume the ambient stratification is not sufficient to prevent the rising plume from reaching the water surface. In the event stratification stops the vertical rise of the plume after, say 30 m, the initial dilutions obtained would be effectively half the above values (approximately calculated from Eqs. 4.33, 4.34, with $d \cong y_{max}$).

For older outfalls which use a single open-ended pipeline, the initial dilutions are usually quite a bit less. As an example, consider a hypothetical outfall discharging 2 m^3/s from a pipe at the bottom depth of 15 m. Equation 4.28 then gives $S \cong 3$.* This is estimated based on the plume rising to the surface and without taking into account the effect of blocking. While these estimates are not necessarily of great accuracy, the marked difference between the

---

* For this case the initial momentum and mass flux would undoubtedly be important. For horizontal buoyant jets see Brooks (1980) or Fischer, et al. (1979).

initial dilutions obtainable from multiport diffusers in relatively deep water compared with single point discharges should be obvious. Whereas the former are typically several hundred, the latter are often less than ten.

For ready reference, Figures 4.5 to 4.7 are included here. These are graphical representations of equations 4.34, 4.27, and the combination when 4.27 is substituted into 4.34, respectively (density difference between effluent and seawater assumed to be 0.025 gram per cubic centimeter (gm/cc)). From these, the order of magnitude of dilutions obtainable from multiple port diffusers can be readily gauged. The accuracy of the analysis and the graphs is probably only ± 20 percent because of the assumptions involved.

The analyses presented in this section pertain to the discharge of wastewater effluent into the ocean from long multiple-port diffusers located in relatively deep water. In particular, it assumes that the flow rate and water depth are such that the diluting water can approach the discharge structure relatively unimpeded. This is the case for all major ocean discharges of sewage effluent. However, if the flow rate per unit length of diffuser is large and the ocean relatively shallow, such as in cooling water discharge from power plants, the above can no longer be applied because the entire water column around the discharge structure may be actively involved in the plume. Analysis of the latter type of mixing is not treated herein. The reader is referred to Jirka and Harleman (1973) and Jirka et al. (1975) for detailed discussions.

Instantaneous Source

Some waste discharge operations, such as disposal of heavier-than-seawater sludge from a hopper barge, are more appropriately simulated as instantaneous sources rather than continuous ones. The discharged sludge can be approximated by a sinking cloud containing both fluid and solids. For small solids, their effect on the physical behavior is primarily the contribution to an increase in the bulk density. As the cloud descends due to its submerged weight (negative buoyancy), it mixes with the ambient seawater. The basic physical phenomenon during this convective descent phase is similar to the rise of thermals in

Figure 4.5 Initial average dilution for an outfall diffuser based on line plume approximation. Graph of Eq. 4.34 (density difference assumed = 0.025 gm/cc).

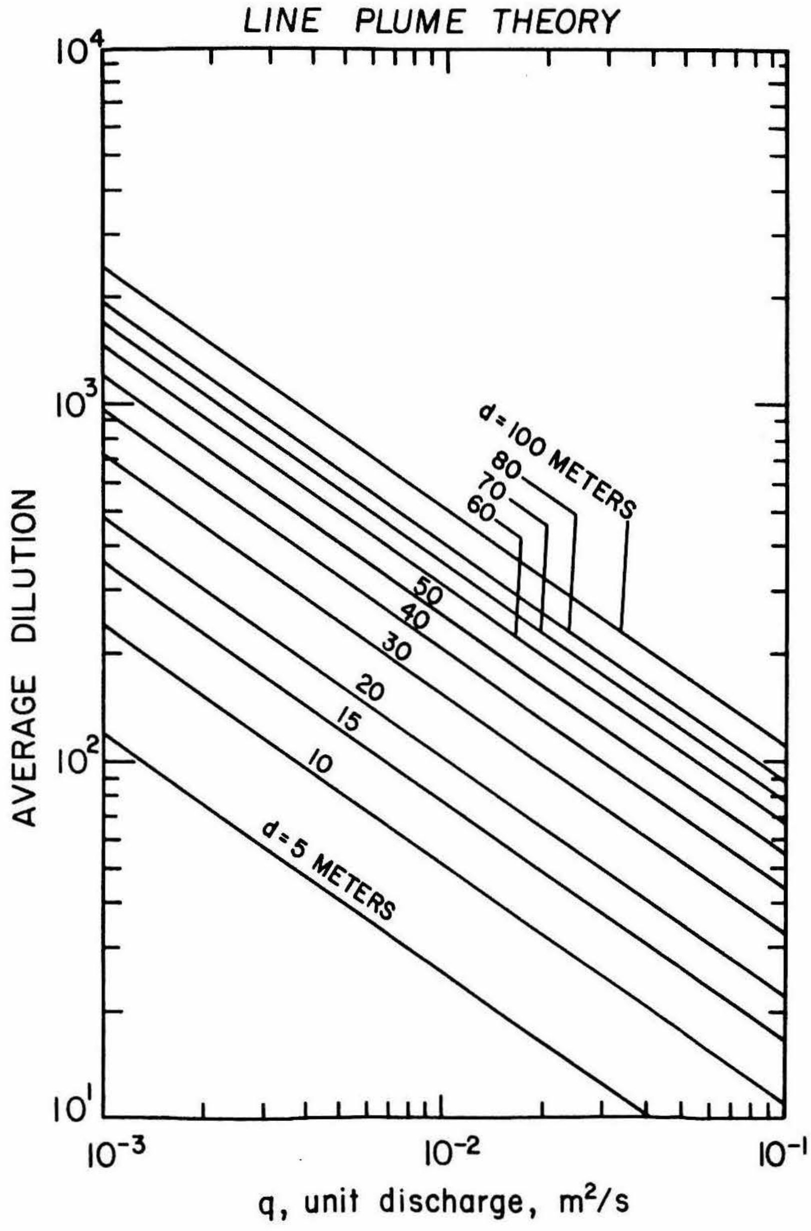


Figure 4.6 Maximum height of rise of wastewater plume from outfall diffuser in linearly stratified ocean based on line plume approximation. Graph of Eq. 4.27 (density difference assumed = 0.025 gm/cc).

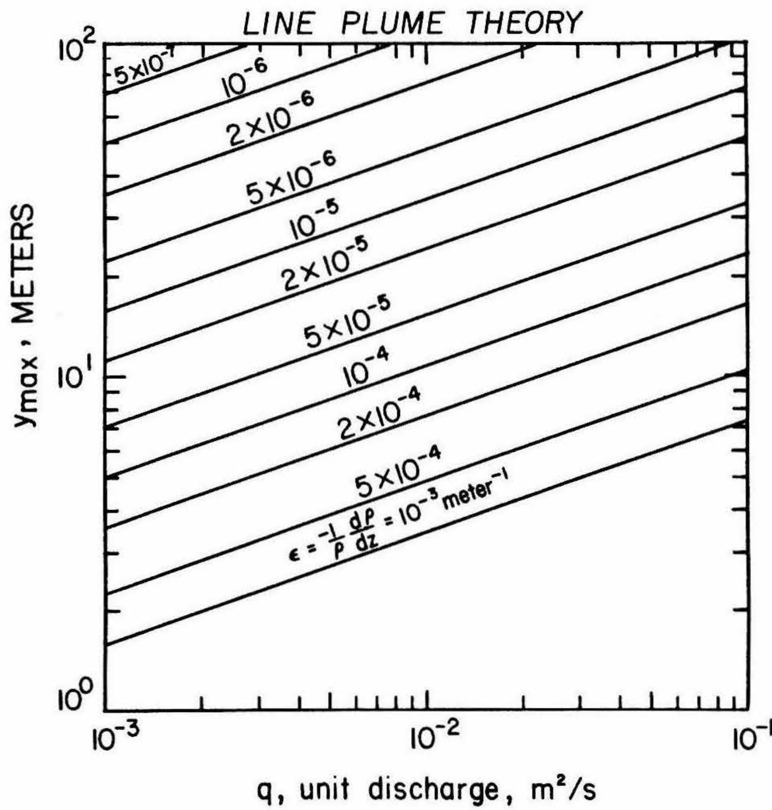
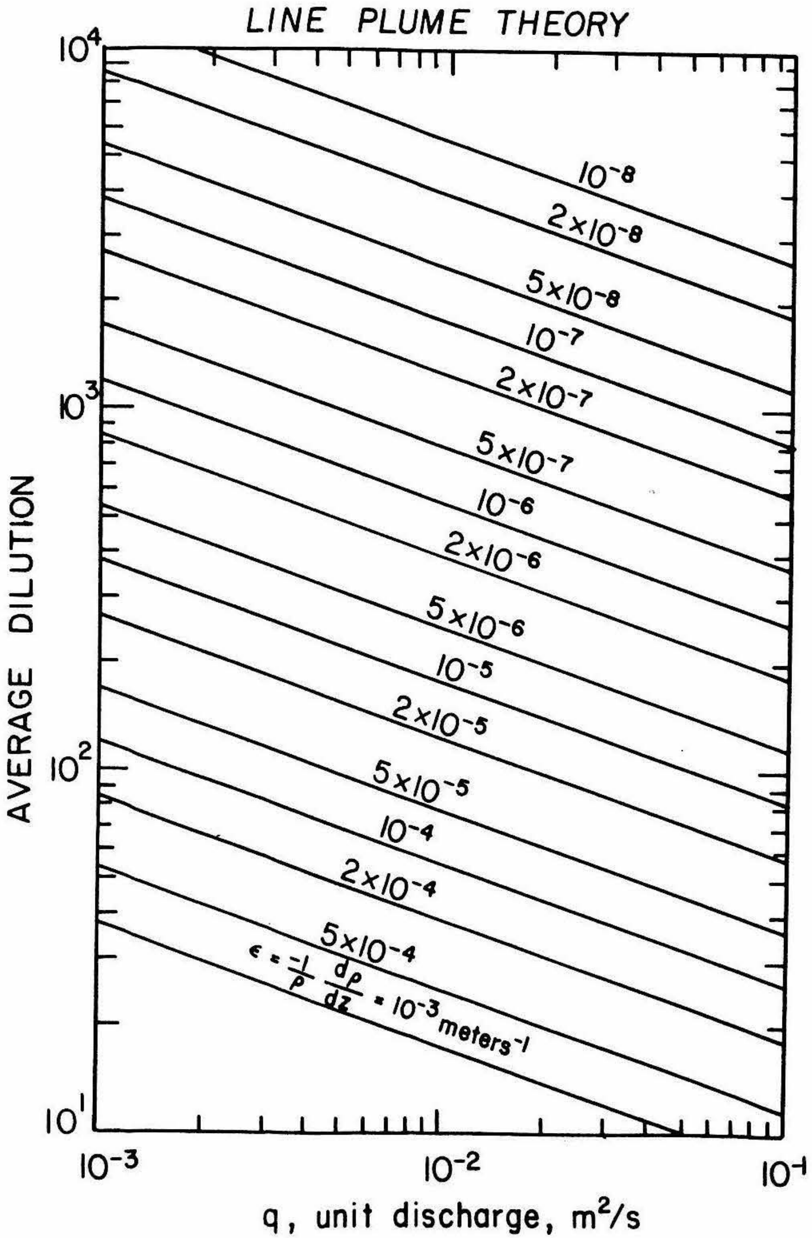


Figure 4.7 Initial average dilution at maximum height of rise for outfall diffuser in linearly stratified ocean. Graph of combined Eq. 4.34 and 4.27 (density difference assumed = 0.025 gm/cc).



the atmosphere. The flow field in and around a thermal (Scorer, 1957) can be summarized as in Figure 4.8. (For a sinking sludge cloud, the direction of travel should be reversed.) The subject is summarized in Turner (1973).

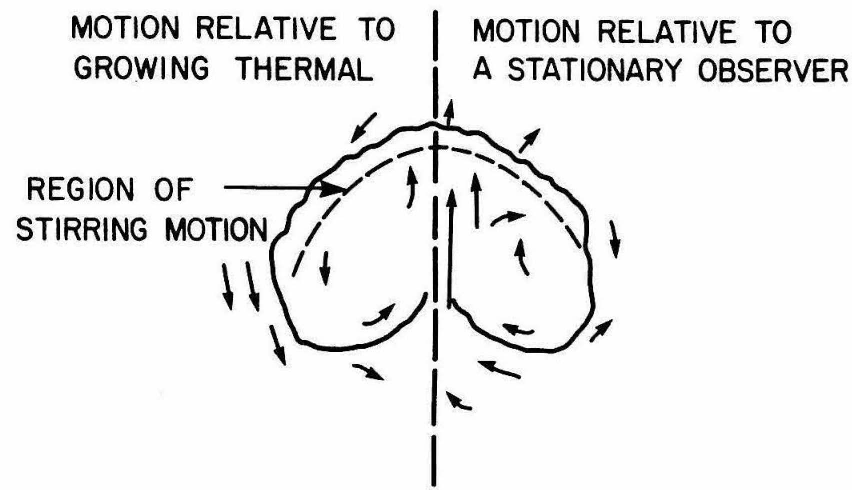


Figure 4.8 Flow field around thermal.

Based on the entrainment concept and similarity assumption, Koh and Fan (1968) developed a calculation procedure for the Navy to estimate the distribution of radioactive debris following deep underwater nuclear explosions. Their model was used by Clark et al. (1971) in assessing the impact of barged waste disposal. The model was later extended by Koh and Chang (1973) to apply specifically for barged disposal. Several disposal methods were considered and the model includes separate calculations for the fluid portion as well as several size fractions of the solid components.

In the following, the simple case of the downward motion of an isolated sludge mass will be examined in a density stratified ocean. It will be assumed that the shape of the cloud as well as the velocity and concentration distributions within the cloud are similar as the cloud descends. Let b, v, c, and ρ denote the half width, average velocity, average concentration, and average density in the cloud. Assume the shape of the cloud can be approximated by a hemisphere. The equations of motion considering the cloud as an entity can be written approximately as

$$\frac{d}{dt}\left(\frac{2\pi}{3} b^3 \rho\right) = \rho_a E \tag{4.35}$$

$$\frac{d}{dt}\left[C_M \frac{2\pi}{3} b^3 \rho u\right] = \frac{2\pi}{3} b^3 g\,(\rho - \rho_a) - F_D \tag{4.36}$$

$$\frac{d}{dt}\left[(\rho - \rho_1)\frac{2\pi}{3} b^3\right] = (\rho_a - \rho_1)\,E \tag{4.37}$$

where E is rate of volume entrainment into the cloud, F_D is a drag force on the cloud, and C_M is the apparent mass coefficient. ρ_a is the density of the ambient water, ρ_1 is a reference density and t is time. Due to the fact that the cloud is an isolated mass moving in the ambient water, hydrostatic pressure distribution (assumed in the continuous source case) may not be realized. The drag term F_D is introduced to account for the overall effect of the nonhydrostatic pressure distribution around the cloud. The value of C_M for a solid sphere is 3/2. One therefore expects $1 \leq C_M \leq 1.5$. For F_D, a standard drag term of the form

$$F_D = \frac{1}{2} C_D\, \rho u^2\, \pi b^2 \tag{4.38}$$

might be adopted. For the entrainment rate E, in analogy with the continuous source, one might write

$$E = 2\pi b^2 \alpha u \tag{4.39}$$

where α is an entrainment coefficient, normally assumed constant. In addition to the above equation, an equation is also needed which expresses the kinematic relation

$$\frac{dz}{dt} = u \tag{4.40}$$

where z is the vertical coordinate, positive downward.

The conservation of buoyancy equation can be written using the conservation of mass equation as

$$\frac{d}{dt}\left[(\rho - \rho_a + \rho_a - \rho_1)\frac{2\pi b^3}{3}\right] = (\rho_a - \rho_1)E = \left(\frac{\rho_a - \rho_1}{\rho_a}\right)\frac{d}{dt}\left[\frac{2\pi}{3} b^3 \rho\right] \tag{4.41}$$

Invoking the Boussinesq assumption,

$$\frac{d}{dt}\left[(\rho - \rho_a)\frac{2\pi b^3}{3}\right] + \frac{2\pi b^3}{3}\frac{d\rho_a}{dz}\,u = 0 \tag{4.42}$$

For simplicity, assume $C_D = 0$ and $C_M = 1$ and let

$$\Delta = g\,\frac{\rho - \rho_a}{\rho_1} \tag{4.43}$$

then

$$\frac{d}{dz}\left(\frac{2\pi}{3} b^3\right) = 2\rho b^2 \alpha \tag{4.44}$$

$$u\frac{d}{dz}\left(\frac{2\pi b^3}{3} u\right) = \frac{2\pi b^3}{3} gc \tag{4.45}$$

$$u\frac{d}{dz}\left(\frac{2\pi}{3} b^3 \Delta\right) = -u\,\frac{2\pi b^3}{3}\, g\left(\frac{d\rho_a}{dz}\right) \tag{4.46}$$

which reduces to

$$\frac{db}{dz} = \alpha \tag{4.47}$$

$$\frac{d}{dz}(b^3 u) = \frac{g\Delta b^3}{u} \tag{4.48}$$

$$\frac{d}{dz}(b^3 \Delta) = -b^3 G \tag{4.49}$$

where $G = \frac{g}{\rho_1}\frac{d\rho_a}{dz}$ which is positive since z is positive downward.

Under the assumption that α is a constant, the mass conservation equation can be integrated at once to yield

$$b = \alpha z + b_o \tag{4.50}$$

where b_o is the value of b at z = 0. Hence the cloud increases in radius linearly in distance. The concentration of any conservative tracer which is absent in the ambient water, therefore, must be

$$c = \frac{c_o b_o^3}{(b_o + \alpha z)^3} \tag{4.51}$$

The dilution S then, can be written

$$S = \frac{c_o}{c} = \frac{(b_o + \alpha z)^3}{b_o^3} \tag{4.52}$$

For large distances compared with b_o ($z/b_o \gg 1$),

$$S = \left(\frac{\alpha z}{b_o}\right)^3 \tag{4.53}$$

which increases very rapidly with distance travelled.

Note that these results (equations 4.50 to 4.52) are independent of the ambient density stratification, being dependent only on the assumption of similarity and entrainment relation. Laboratory experiments (e.g., Scorer, 1957; Richards, 1961) showed that the linear growth of the cloud is observed. However, the growth rate appears to be variable among seemingly identical runs. In other words, α, the entrainment coefficient is a constant for each individual experiment but varies between experiments. Turner (1960), in investigating the dynamics of buoyant vortex rings, found by assuming similarity that

$$\alpha = \frac{B}{2\pi g\ c_1 K^2} \tag{4.54}$$

where B and K are buoyancy and vorticity in the vortex ring. c_1 was found to be about 0.16. When waste material is dumped from a barge, vorticity can be generated from the initial momentum and buoyancy.

However, as the cloud descends in a stratified ocean, the vorticity decreases and approaches zero so that the relation cannot hold.

Experimental data on the entrainment coefficient for the buoyant element shows a relatively large degree of scatter varying generally between 1/7 and 1/3. The effect of this more than two-fold range in α on the dilution is quite significant. For our purpose in this chapter, the average value $\alpha = 0.25$ as found by Scorer (1957) will be adopted for definiteness.

Return now to equations 4.47 to 4.49 and consider the case G = constant. Let the initial conditions be $u = u_o$, $b = b_o$, $\Delta = \Delta_o$ at $z = 0$. Then the solution is given by

$$b = b_o + \alpha z \tag{4.55}$$

$$\Delta = \frac{\Delta_o b_o^3}{b^3}\left\{1 - \frac{H}{4\alpha}\left(\frac{b^4}{b_o^4} - 1\right)\right\} \tag{4.56}$$

$$u = \frac{u_o b_o^3}{b^3}\left\{1 + \frac{2}{4\alpha F^2}\left[\left(1 + \frac{H}{4\alpha}\right)\left(\frac{b^4}{b_o^4} - 1\right) - \frac{H}{8\alpha}\left(\frac{b^8}{b_o^8} - 1\right)\right]\right\}^{1/2} \tag{4.57}$$

where

$$H = \frac{Gb_o}{\Delta_o} \quad \text{and} \quad F = \frac{u_o}{\sqrt{\Delta_o b_o}} \tag{4.58}$$

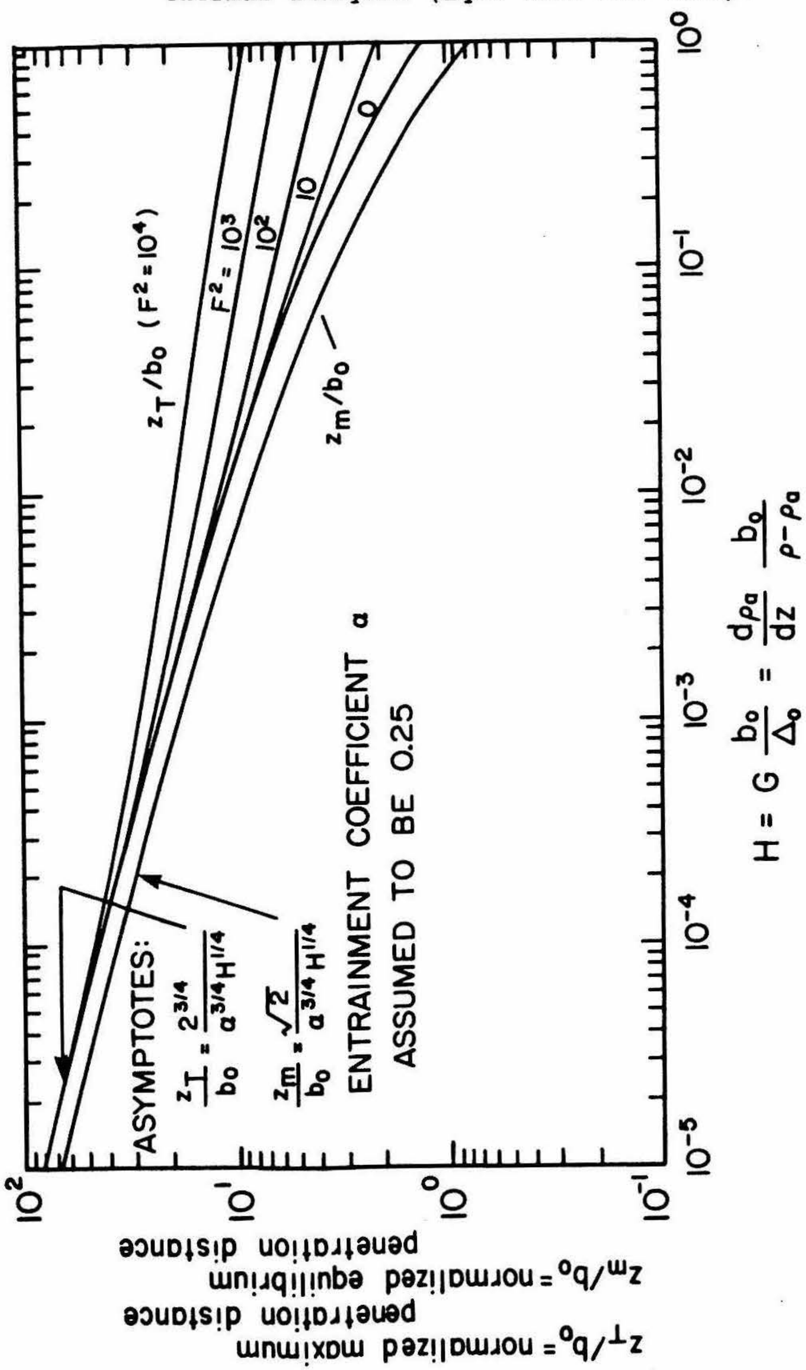
Two interesting features of the solution can be deduced from this. Two distances of fall may be defined: the first will be denoted by z_m which is the depth at which $\Delta=0$; the second will be denoted by z_T which is the depth at which $u=0$. The final depth at which the waste cloud resides will be in between these two depths. These depths can be obtained from the solution to be

$$z_m = \frac{b_o}{\alpha}\left[\left(1 + \frac{4\alpha}{H}\right)^{1/4} - 1)\right] \tag{4.59}$$

$$z_T = \frac{b_o}{\alpha}\left\{\left[1 + \frac{4\alpha}{H}\left(1 + \sqrt{1 + HF^2}\right)\right]^{1/4} - 1\right\} \tag{4.60}$$

Equations 4.59 and 4.60 are shown plotted in Figure 4.9.

Figure 4.9 Normalized penetration distances for instantaneous barge dump based on buoyant thermal analysis (Eqs. 4.59 and 4.60).



Recall that dilution is given by

$$\frac{c_o}{c} = \left(\frac{b_o + \alpha z}{b_o}\right)^3 = \left(1 + \alpha \frac{z}{b_o}\right)^3 \tag{4.61}$$

Thus if we use z_T as the value of z to estimate dilution,

$$S = \frac{c_o}{c} = \{1 + \frac{4\alpha}{H}(1 + \sqrt{1 + HF^2})\}^{3/4} \tag{4.62}$$

This is shown plotted in Figure 4.10.

As an example consider the case when a load of sludge with bulk density 1.03 gm/cc is discharged downward from a hopper barge in an ocean depth of 15 m. Let the ocean water density be 1.024 and the ambient density gradient be 10^{-5} m^{-1}. Assume no appreciable initial momentum and let the radius of the cloud be 5 m. Then H= $10^{-5}/.006 \times 5 = 3.3 \times 10^{-4}$. From Figure 4.9, the cloud would reach the bottom. At that point, the dilution would be approximately

$$\left(1 + \frac{1}{4} \cdot \frac{15}{5}\right)^3 = 5$$

For a smaller load, say, with radius 1 m, H = $10^{-5}/.006 = 1.67 \times 10^{-3}$ and $z_T/b_o \simeq 20$, so that it would still reach the bottom. In this case, the dilution would be approximately

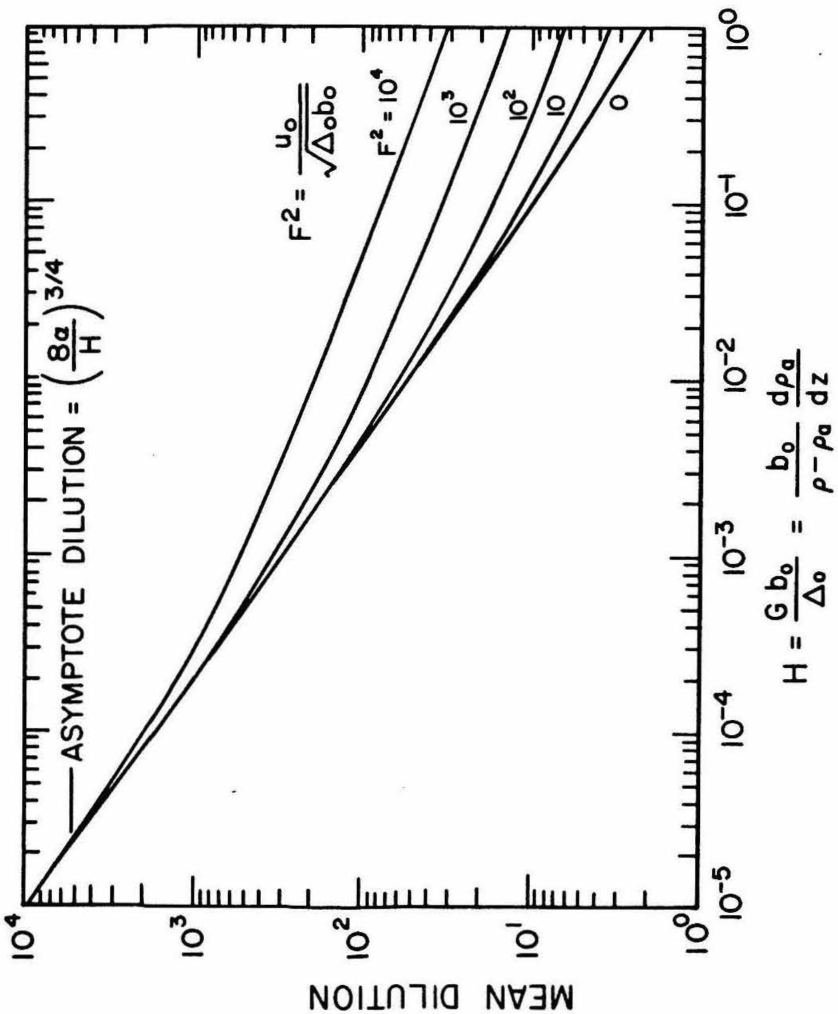
$$\left(1 + \frac{1}{4} \cdot \frac{15}{1}\right)^3 \simeq 100$$

It is thus seen that the initial dilution obtainable from instantaneous discharges of heavier-than-seawater sludge from hopper barges depends very much on the size of the dump.

OCEAN ENVIRONMENTAL CONDITIONS

The physical fate of the discharged waste material (both initial dilution and subsequent mixing and transport) depends not only on the characteristics of the material itself, the mode of discharge, but also the ocean environmental conditions. This chapter has concentrated thus far on the first two aspects. In this section, we shall discuss those ocean environmental conditions which most influence the initial phase of the mixing process.

Figure 4.10 Dilutions at maximum penetration depth z_T (Eq. 4.62) for instantaneous dump.



Density Stratification

One of the most important ocean environmental conditions which influences the initial mixing processes is the density stratification. (Figures 4.11 and 4.12 show the density profiles chosen for the design of the Sand Island outfall and the resulting dilutions respectively.) The ocean is often stratified in density due either to temperature or salinity variations. Due to the large dilutions obtainable from a multiport diffuser within a short distance of travel of the discharged waste material, the presence of an ambient density stratification frequently results in the waste field terminating its vertical motion before reaching the surface. In the development of the analyses presented in previous sections of this chapter, the ambient density profile has been incorporated as part of the formulation. While solutions were shown only for simple forms of this variable (e.g., linear profile), there is no inherent limitation in the formulation.

It is noted that the important quantity is not so much the density itself but the density gradient. In fact throughout this chapter, it has been assumed that density changes are small and density variation need be considered only if it occurs as a difference. Because of this, determinations of density must be made accurately so that when differences are taken, sufficient precision is retained. Measurements of ocean density profiles are made indirectly by measuring instead the temperature and salinity and then using an empirical relation to determine density. Hydrographic tables (such as US Navy Hydrographic Office Pub. 615, 1952) are available for this purpose. A more concise table for typical ocean salinities can be found in Appendix A of Fischer et al. (1979).

The density profile in the ocean varies with both location and time. The temporal variations consist of both seasonal and diurnal components. This implies that evaluations of dilution should be made for a variety of density stratifications. For each evaluation, it may be assumed that the ambient density is constant with time. This is because the time scale of vertical motion of the waste material is relatively short (being on the order of minutes) compared with that for material changes in the density stratification. Representative density profiles may be constructed by averaging several measured profiles

Figure 4.11 Density profiles chosen for design calculations for Sand Island Outfall, Honolulu, Hawaii (R.M. Towill Corp., 1972).

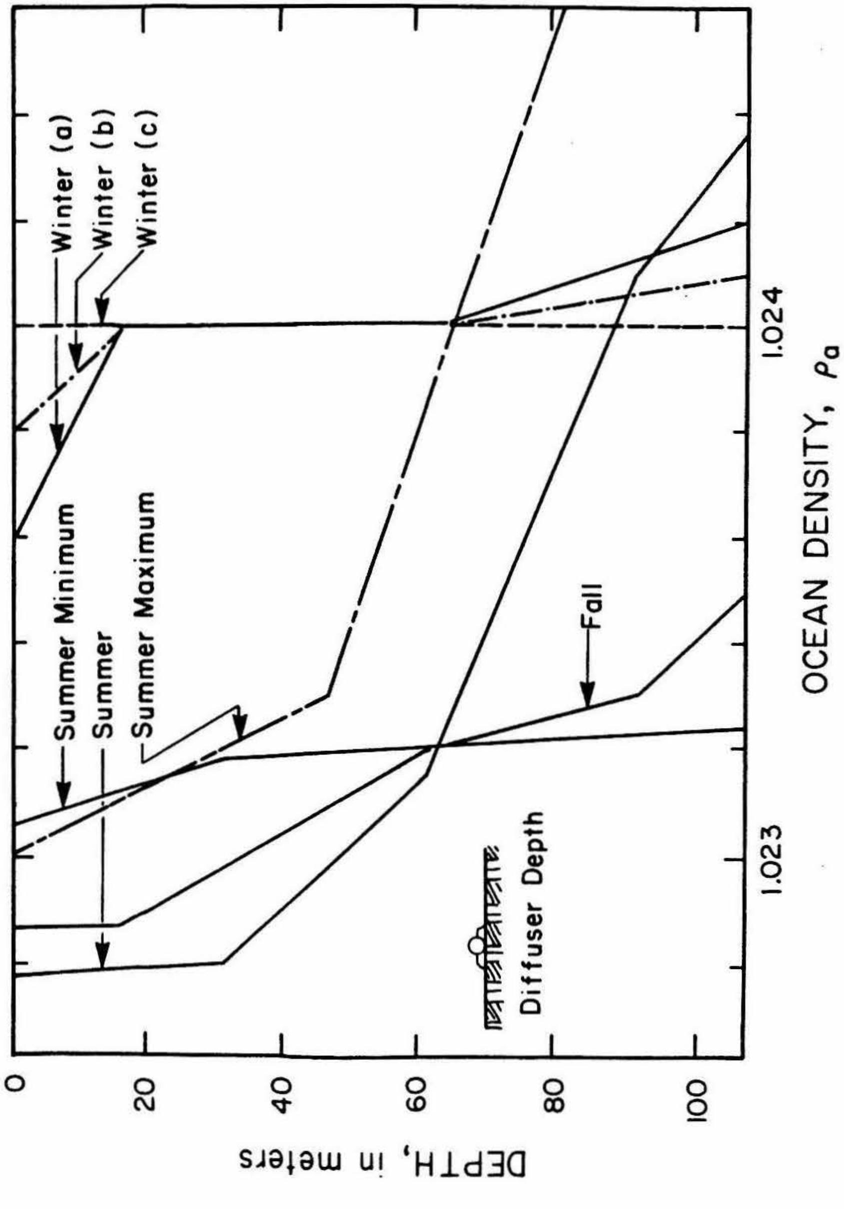
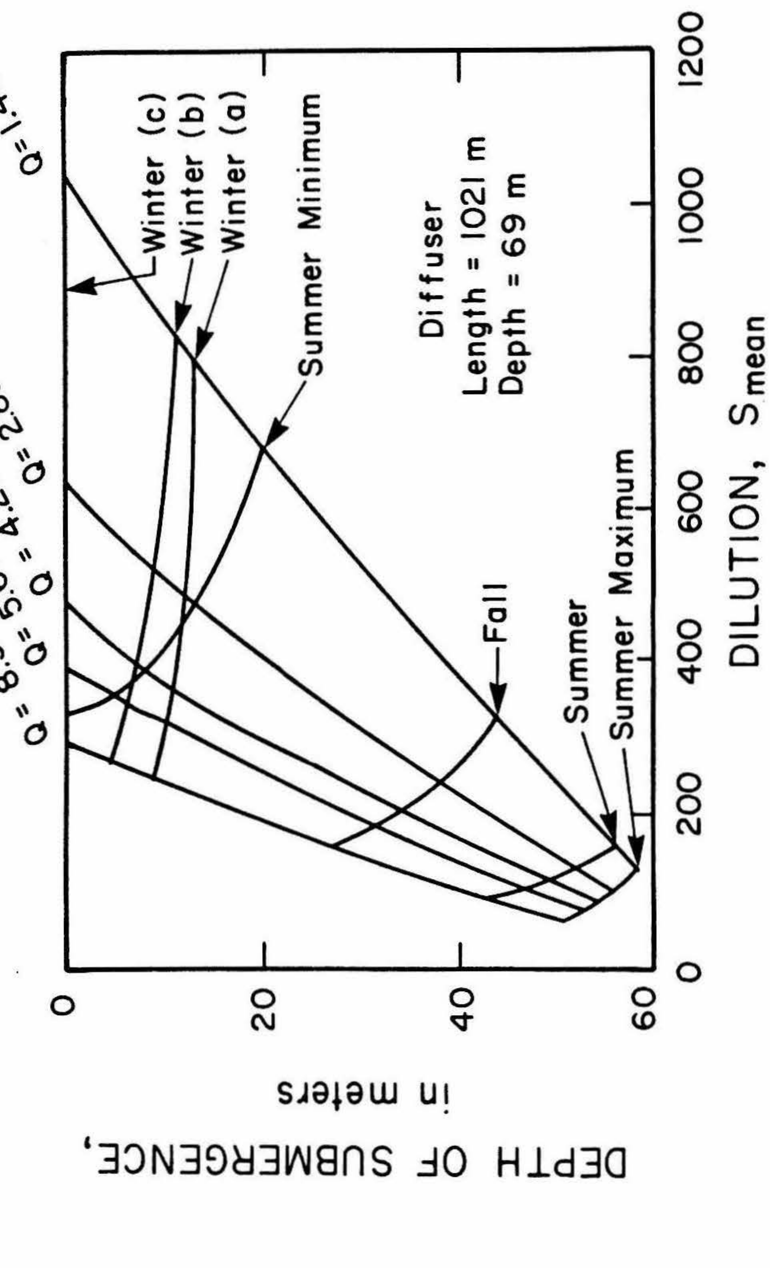


Figure 4.12 Dilutions and submergence of wastewater field for Sand Island Outfall, Honolulu, Hawaii (R.M. Towill Corp., 1972).



for each period of a year. Design of outfalls often involves evaluation of initial dilution under these conditions.

Ocean Currents

Ocean currents are another environmental variable which influence the mixing processes. A current will tend to bend the plume trajectory from an outfall and thus increase its travel length and promote larger dilutions for the same vertical travel. This results in general in both larger dilutions and lower height of rise. For barge disposal of sludge, the current of interest is the relative velocity between the barge and the ocean water.

Measurements of currents in the ocean are made by means of either current meters or drogues. The former gives an Eulerian time-series of current velocity at fixed locations (lasting typically on the order of a month for each record). The latter can give a Lagrangian view of the water movement (however, the duration of the measurements are limited by the ability to follow the drogues, typically less than one day). Both types of data are useful for evaluation of waste disposal consequences. Figure 4.13 illustrates some current meter data taken off southern California. It is possible to simulate the movement of a hypothetical drogue from current meter data if it is assumed that the spatial correlation is perfect. This is done by integrating the current meter data. The result is known as a progressive vector diagram as illustrated in Figure 4.14. For continuous discharge, it is also useful to construct the streakline from the current time series by reversing the data string and integrating. The streakline simulates the plume centerline observed at the final time of the current sequence again assuming perfect spatial correlation. This is shown in Figure 4.15. This type of display permits a clearer visualization of the water movement than the simple plot shown in Figure 4.13.

The behavior of a single round buoyant jet in a cross current has been investigated using the integral approach by Abraham (1970), Hirst (1971), among others. Near the source, the behavior is dominated by the buoyant jet. As the jet bends over, it gradually takes on the characteristics of a line thermal. Wright (1977) studied the same problem in the laboratory for the case when the ambient is both flowing and density stratified. Roberts (1977)

Figure 4.13 Measured (solid) and low-pass filtered (dashed) currents near Orange County Outfall.

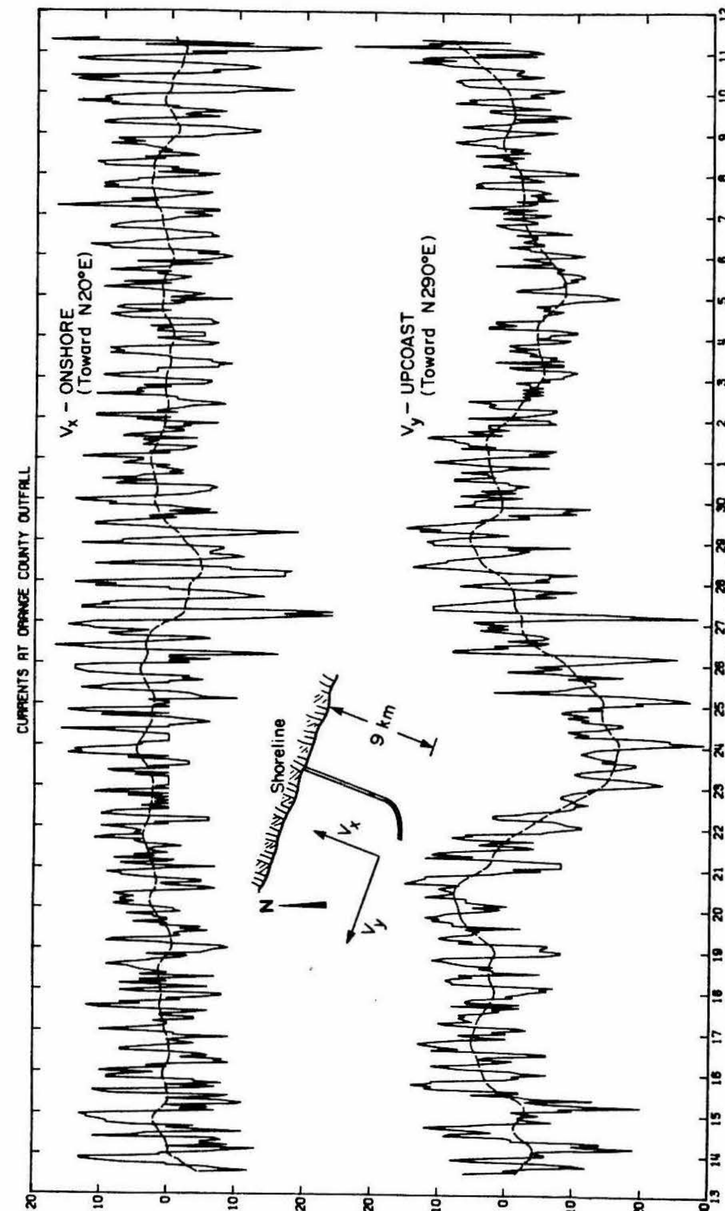


Figure 4.14 Progressive vector diagram for currents shown in Fig. 4.13.

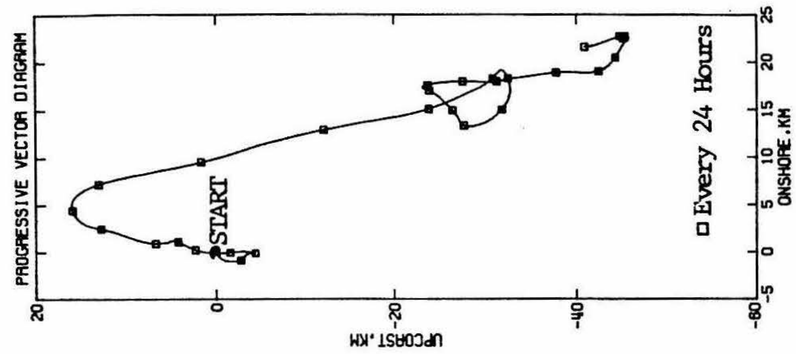
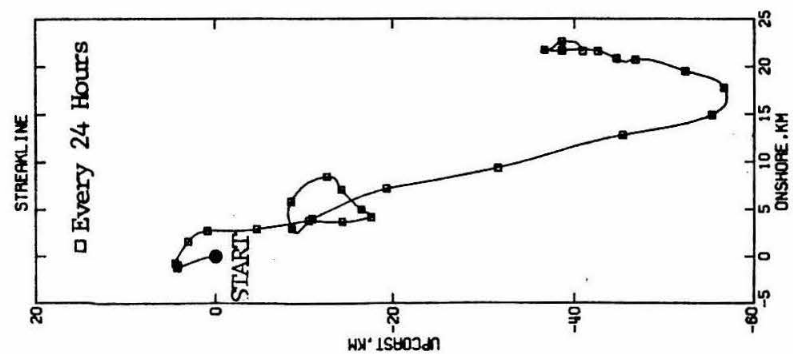


Figure 4.15 Streakline for currents shown in Fig. 4.13.



investigated the effect of an ocean current on the mixing resulting from a long diffuser oriented at various angles to the flow. In all cases, initial dilution is larger when there is a current than when the ambient water is stationary.

In the philosophy of ocean wastewater discharge from outfalls it is implicit that there is sufficient through flow in the region to transport the continuous discharge away from the site. While this may be a good assumption for discharge into open coastal areas, it is by no means clear for discharge into semi-enclosed bodies of water such as estuaries. In these cases, it is important to critically examine the transport and mixing in the zone beyond the initial dilution region, since the diluting water may already contain a certain concentration of waste. Let c_o be the concentration of wastewater in the ambient fluid and let S be the initial dilution. Thus one part of effluent is mixed with S-1 parts of diluting water which iself has a concentration of c_o. The concentration of wastewater in the mixture c is then

$$c = \frac{1 + (S - 1)\, c_o}{S} = \frac{1}{S} + c_o - \frac{c_o}{S}$$

so that the net dilution, interpreted as the inverse of the final concentration is

$$S_{net} = \frac{1}{c} = \frac{1}{\frac{1}{S} + \frac{1}{S_o} - \frac{1}{SS_o}} = \frac{SS_o}{S_o + S - 1}$$

One may call S_{net}, S, and S_o the net initial dilution, the near-field dilution and the far-field dilution, respectively.

SUMMARY

In this chapter, the concept of initial dilution is discussed. It is pointed out that initial dilution is only rather loosely definable, being that part of the dilution resulting from the mixing process dominated by the buoyancy and momentum of the discharge.

Two types of delivery systems are considered in this chapter: outfalls and vessels. Simple results are presented which permit rapid estimation of initial dilutions for simple ambient ocean conditions. The initial dilutions obtainable for typical large outfalls equipped with long multiport diffusers discharging in relatively deep water is on the order of 10^2 to 10^3. For outfalls without diffusers, usually discharging in shallower water, they are often much less, being on the order of ten. For heavier-than-seawater waste discharged instantaneously from barges, the initial dilutions obtainable are sensitive to the size of each individual dump and can vary substantially. If barges are used to transport the waste to deep water and the material released in many smaller dumps instead of a few large ones, the dilution can be increased substantially.

REFERENCES

Abraham, G. 1970. The flow of round buoyant jets issuing vertically into ambient fluid flowing in a horizontal direction. Advan. Water Pollut. Res. Proc. Int. Conf. Water Poll. Res. 5th Paper III-15.

Baumgartner, D.J., D.S. Trent, and K.V. Byram. 1971. User's guide and documentation for outfall plume model. EPA: Working Paper #80, May 1971.

Brooks, N.H. 1973. Dispersion in hydrologic and coastal environments. EPA, Rep. 660/3-73-010. (Also, CA Inst. Tech., W.M. Keck Lab. Rep. KH-R-29.)

Brooks, N.H. 1980. Synthesis of stratified flow phenomena for design of outfalls. Second International Symposium on Stratified Flows, June 1980.

Chen, J.C. and E.J. List. 1976. Spreading of buoyant discharges. Proc. Int. Seminar on Turbulent Buoyant Convection. Dubrovnik, Yugoslavia, August 30-September 4, 1976.

Chen, J.C. 1980. Studies on gravitational spreading currents. Ph.D. Thesis. Pasadena: CA Inst. Tech., W.M. Keck Lab.

Clark, B.D., W.F. Rittal, D.J. Baumgartner, and K.V. Byram. 1971. The barged ocean disposal of wastes. EPA Report, July 1971.

Ditmars, J.D. 1969. Computer program for round buoyant jets into stratified ambient environments. Pasadena: CA Inst. Tech., W.M. Keck Lab. Tech. Memo. 69-1.

Fan, L. N. 1967. Turbulent buoyant jets into stratified or flowing ambient fluids. Pasadena: CA Inst. Tech., W.M. Keck Lab. Rep. KH-R-15.

Fischer, H.B., E.J. List, R.C.Y. Koh, J. Imberger, and N.H. Brooks. 1979. Mixing in inland and coastal waters. NY: Academic Press.

Fox, D.G. 1970. Forced plume in a stratified fluid. J. Geophys. Res. 75:6818-35.

George, W.K., R.L. Alpert, F. Tamanini. 1977. Turbulence measurements in an axisymmetric buoyant plume. Int. J. Heat Mass Transfer, 20:1145-1154.

Gross, M.G. 1976. Waste disposal. MESA New York Bight Atlas Monograph 26. Albany: New York Sea Grant Institute, July 1976.

Hirst, E.A. 1971. Analysis of round, turbulent, buoyant jets discharged to flowing stratified ambients. Oak Ridge Nat. Lab. Rep. ORNL-4685.

Jirka, G., D.R.F. Harleman. 1973. The mechanics of submerged multiport diffusers for buoyant discharges in shallow water. Cambridge: MA Inst. Tech., Ralph M. Parsons Lab. Rep. 169.

Jirka, G.H., G. Abraham, D.R.F. Harleman. 1975. An assessment of techniques for hydrothermal prediction. Cambridge: MA Inst. Tech., Ralph M. Parsons Lab. Rep. 203. July 1975.

Koh, R.C.Y. and L.N. Fan. 1968. Prediction of the radioactive debris distribution subsequent to a deep underwater nuclear explosion. Pasadena, CA: Tetra Tech Rep. No. 129, Oct. 1968.

Koh, R.C.Y. and L.N. Fan. 1970. Mathematical models for the prediction of temperature distributions resulting from the discharge of heated water into large bodies of water. EPA Rep. 16130 DWO 10/70. (Also, Tetra Tech, Inc. Rep. TC-170.)

Koh, R.C.Y. and Y.C. Chang. 1973. Mathematical model for barged ocean disposal of wastes. EPA-660/2-73-029. December, 1973.

Koh, R.C.Y. and N.H. Brooks. 1975. Fluid mechanics of wastewater disposal in the ocean. Annual Review of Fluid Mechanics, 7:187-211.

Koh, R.C.Y. 1976. Buoyancy-driven gravitational spreading. Proc. 15th Int. Conf. on Coastal Engineering, Honolulu, Hawaii, July 11-17, 1976.

Kotsovinos, N.E. 1975. A study of the entrainment and turbulence in a plane buoyant jet. Pasadena: CA Inst. Tech., W.M. Keck Lab. Rep. KH-R-32, August 1975.

List, E.J. and J. Imberger. 1973. Turbulent entrainment in buoyant jets and plumes. J. Hydraul. Div. Proc. ASCE 99:1462-74.

Morton, B. R., G.I. Taylor, J.S. Turner. 1956. Turbulent gravitational convection from maintained and instantaneous sources. Proc. Roy. Soc. London Ser. A. 234:1-23.

Mueller, J.A. and A.R. Anderson. 1978. Industrial wastes. MESA New York Bight Atlas Monograph 39, Albany: New York Sea Grant Institute, January 1978.

Rawn, A.M. and H.K. Palmer. 1930. Pre-determinng the extent of a sewage field in sea water. Trans. ASCE, 94:1036-1060.

Richards, J.M. 1961. Experiments on the penetration of an interface by buoyant thermals. J. Fluid Mech. 11:369.

Roberts, P.J.W. 1977. Dispersion of buoyant wastewater discharged from outfall diffusers of finite length. Pasadena: CA Inst. Tech., W.M. Keck Lab. Rep. KH-R-35.

Rouse, H., C.S. Yih and H.W. Humphreys. 1952. Gravitational convection from a boundary source. Tellus. 4:201-210.

Scorer, R.S. 1957. Experiments on convection of isolated masses of buoyant fluid. J. Fluid Mech. 2:583.

Sharp, J.J. 1969. Spread of buoyant jets at the free surface. J. Hydraul. Div. Proc. ASCE. 95:1771-1773.

Sotil, C.A. 1971. Computer program for slot buoyant jets into stratified ambient environments. Pasadena: CA Inst. Tech., W.M. Keck Lab. Tech Memo. 71-2.

R.M. Towil Corp. 1972. Final design report Sand Island ocean outfall system, city and county of Honolulu.

Turner, J. S. 1973. Buoyancy effects in fluids. Cambridge, England: Cambridge University Press.

U.S. Navy Hydrographic Office. 1952. Tables for seawater density. Washinton, D.C.: H.O. Pub. No. 615.

Wright, S.J. 1977. Effects of ambient crossflows and density stratification on the characteristic behavior of round turbulent buoyant jets. Pasadena: CA Inst. Tech., W.M. Keck Lab. Tech Rep. KH-R-36.